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Flow equations and normal ordering: a survey

Franz Wegner

Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 19, D-69120 Heidelberg, Germany

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Abstract

First we give an introduction to the method of diagonalizing or block-diagonalizing continuously a Hamiltonian and explain how this procedure can be used to analyse the two-dimensional Hubbard model. Then we give a short survey on applications of this flow equation on other models. Finally we outline, how symmetry breaking can be introduced by means of a symmetry breaking of the normal ordering, not of the Hamiltonian.

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1. Introduction

Renormalization group plays an important role both in high-energy physics and in condensed matter physics. It was invented in high-energy physics [1, 2] in order to resolve the problems with divergencies at large momenta. It was developed as a tool to define how an interaction with cut-off at some momentum Λ has to vary as Λ increases *up to* the limit infinity so that the expectation values converge to a finite limit.

For a long time, the explanation of critical behaviour was an open question in condensed matter physics. Wilson [4] showed along ideas developed by Kadanoff [3] that also this problem is solved by the renormalization group. In condensed matter, most often one does not worry about large momenta, in particular, if one considers a system on a lattice where momenta are restricted to the Brillouin zone. Then the problem of critical phenomena resides in small momenta and one has to integrate out the contributions of the interaction at momenta *down to* some small cut-off Λ and finally to consider the limit where this cut-off approaches zero. This applies for bosonic degrees of freedom which have the advantage that their quantum nature is normally irrelevant so that they can be considered as classical fields.

The situation is different for fermionic degrees of freedom. They do not show a classical limit. Therefore, other methods had to be developed. The essential physics in a condensed fermionic system comes from the behaviour at the Fermi edge. Therefore, it is natural to focus on the behaviour close to the Fermi edge and to eliminate the degrees of freedom away from this Fermi edge so that the cut-off describes now the distance from the Fermi edge. This procedure was pushed forward by Shankar [5, 6]. It has been applied to fermionic systems among others by Zanchi and Schulz [7], Salmhofer and Honerkamp [9], Halboth and Metzner [8].

There is a second approach which was started in 1993/1994. It appeared under the name of *Flow equations for Hamiltonians* [12] and *Similarity transformation* [10, 11] and is also often called *Continuous unitary transformation* (CUT). Głazek and Wilson and myself were at that time unaware that mathematicians in the field of control theory had developed similar ideas under the names *Double Bracket Flow* [14] and *Isospectral Flow* [13, 15]. The basic idea is to choose the Hamiltonian in a certain basis, e.g. the basis of Bloch waves and then to eliminate first the off-diagonal matrix elements between states which differ strongly in energy. This is done by a continuous unitary transformation as function of a flow-parameter ℓ. We start from the initial Hamiltonian $H = H(0)$ and obtain a Hamiltonian $H(\ell)$ by means of a unitary transformation $U(\ell)$,

$$H(\ell) = U(\ell)HU^{\dagger}(\ell). \tag{1}$$

Differentiation with respect to ℓ yields

$$\frac{\mathrm{d}H(\ell)}{\mathrm{d}\ell} = [\eta(\ell), H(\ell)] \tag{2}$$

with the generator η of the unitary transformation

$$\eta(\ell) = \frac{\mathrm{d}U(\ell)}{\mathrm{d}\ell}U^{\dagger}(\ell) = -\eta^{\dagger}(\ell). \tag{3}$$

This is the flow equation for the Hamiltonian. The main intention of the use of these flow equations by now is not the investigation of critical phenomena, but the diagonalization or block-diagonalization of the Hamiltonians whether they describe elementary particles or strongly correlated solids. The scheme has in common with renormalization in critical phenomena that it focuses on a smaller and smaller region. It means in this case that off-diagonal matrix elements which connect two states with energy difference $\Delta\epsilon$ become small and finally negligible when the flow-parameter ℓ becomes large in comparison to $1/(\Delta\epsilon)^2$.

Obviously $U(\ell)$ or actually $\eta(\ell)$ have to be chosen in an appropriate way. There are various ways of doing so.

2. Various choices for the generator

The most obvious way is to choose a generator which nearly always diagonalizes the Hamiltonian. If one chooses

$$\eta = [H^{\mathrm{d}}, H], \tag{4}$$

where H^{d} is the diagonal part of the Hamiltonian, then we find that the sum of the squares of the off-diagonal matrix elements decays like

$$\sum_{k,k'|k\neq k'} \frac{\partial h_{k,k'}h_{k',k}}{\partial \ell} = -2\sum_{k,k'}(\epsilon_k - \epsilon_{k'})^2 h_{k,k'}h_{k',k}, \tag{5}$$

where the diagonal matrix elements are denoted by ϵ.[1] Thus the off-diagonal matrix elements decay unless (which happens only rarely) the elimination stops with a non-zero off-diagonal element between two degenerate levels[2]. Note, however, that in general also ϵ depends on ℓ.[3]

[1] The flow equation now reads $\mathrm{d}H/\mathrm{d}\ell = [[H^{\mathrm{d}}, H], H]$ which explains the notion double-bracket equation.

[2] These argument as well as the arguments on the cost function below apply rigorously for finite matrices. For the infinite-dimensional Hilbert space they are guidelines.

[3] In the case of the elimination of the electron–phonon coupling these couplings decay even if the states are finally degenerate [16]. Depending on the elimination scheme such off-diagonal matrix elements can survive in the Kondo Hamiltonian below the Kondo temperature [17].

Although it seems quite desirable to diagonalize the Hamiltonian completely, there is the drawback that nearly always approximations have to be introduced. In order that they do not create too large errors, it is advisable to perform only weak unitary transformations. This can be done by bringing the Hamiltonian to a block-diagonal form. In the case of the elimination of the electron–phonon coupling, we eliminate only the matrix elements which change the number of the phonons, which is the electron–phonon coupling. In this case we are left with an effective electron–electron interaction which has the nice properties (i) that it is attractive between all pairs of electrons with total momentum zero in contrast to Fröhlich's Hamiltonian and (ii) it is instantaneous. This second property is inherent to the scheme of flow equations.

Another case where we bring the Hamiltonian in the block-diagonal form is the elimination of the contributions in an electronic system which does not conserve the number of quasi-particles (electrons above and holes below the Fermi edge). Then the eigenstates are states with fixed numbers of these quasi-particles, so that the excitation energies can be read off the one-particle contribution of the effective Hamiltonian and the two-particle excitations are now excitations determined from a two-particle problem [12].

With H consisting of the diagonal and the off-diagonal part $H = H^{\rm d} + H^{\rm r}$, the generator η may be rewritten as

$$\eta = [H, H^{\rm r}]. \tag{6}$$

We are free, however, to modify $H^{\rm r}$. If we choose

$$H^{\rm r} = [N, [N, H]], \tag{7}$$

where N is the particle operator of the phonons or of the quasi-particles, then the particle number violating terms will be eliminated. To see this we may introduce a cost function

$$G(H) = \frac{1}{2}\sum g_{ij,kl} H_{ji} H_{lk} = \frac{1}{2}\,{\rm tr}(H H^{\rm r}), \qquad H^{\rm r}_{ij} = \sum g_{ij,kl} H_{lk} \tag{8}$$

where we require that g is symmetric and the cost function is real,

$$g_{ij,kl} = g_{kl,ji} = g^*_{ji,lk}. \tag{9}$$

Then one obtains

$$\frac{{\rm d}G}{{\rm d}\ell} = {\rm tr}([\eta, H]H^{\rm r}) = {\rm tr}(\eta[H, H^{\rm r}]). \tag{10}$$

If G is semi-positive definite then the choice

$$\eta = [H, H^{\rm r}] \tag{11}$$

yields

$$\frac{{\rm d}G}{{\rm d}\ell} = {\rm tr}([H, H^{\rm r}][H, H^{\rm r}]) \leqslant 0. \tag{12}$$

Note that η is anti-Hermitian. The derivative $\frac{{\rm d}G}{{\rm d}\ell}$ vanishes only, if $H^{\rm r}$ commutes with H. In all other cases the cost function decreases. The choice $H^{\rm r} = [N, [N, H]]$ yields the cost function

$$G = \tfrac{1}{2}\,{\rm tr}([H, N][N, H]) \tag{13}$$

which becomes zero only if H commutes with N.

This allows another procedure useful for systems with symmetry breaking. We may choose quite generally

$$H^{\rm r} = \sum_\alpha [v^\alpha, [v^\alpha, H]]. \tag{14}$$

If v is a one-particle operator

$$v = \sum_k v_k c_k^\dagger c_k, \tag{15}$$

then the evaluation of $[v, [v, H]]$ multiplies terms of type $c_{k_1}^\dagger c_{k_2}^\dagger \ldots c_{q_1} c_{q_2} \ldots$ in H by

$$r_{k_1 k_2 \ldots q_1 q_2 \ldots} = \left(v_{k_1} + v_{k_2} + \cdots - v_{q_1} - v_{q_2} - \cdots\right)^2. \tag{16}$$

This elimination function indicates how urgently we wish to eliminate such terms in the Hamiltonian. We have used this form of H^{r} in order to calculate the effective potential of the Hubbard model described by the Hamiltonian

$$H = -t \sum_{\text{n.n.}} c_{r's}^\dagger c_{rs} - t' \sum_{\text{n.n.n.}} c_{r's}^\dagger c_{rs} + U \sum_r \left(n_{r\uparrow} - \frac{1}{2}\right)\left(n_{r\downarrow} - \frac{1}{2}\right), \tag{17}$$

in second order in the coupling U (weak coupling limit). We used the condition

$$-v_{-k} = v_k = v_{k+q_0} \tag{18}$$

which transformed the Hamiltonian into a molecular-field form for the expected order parameters (superconductivity, antiferromagnetism, flux-phases, Pomeranchuk instability) that is we allowed for non-zero

$$\left\langle c_{ks}^\dagger c_{-ks'}^\dagger \right\rangle, \quad \left\langle c_{ks}^\dagger c_{k+q_0 s'} \right\rangle, \quad \left\langle c_{ks}^\dagger c_{ks'} \right\rangle \quad \text{and even} \quad \left\langle c_{ks}^\dagger c_{-k+q_0 s'}^\dagger \right\rangle. \tag{19}$$

Depending on the couplings the above-mentioned instabilities showed up. The temperature dependence enters via normal ordering. Since the renormalization flow does not only alter two-particle interactions but also generates higher-particle interactions, the two-particle interaction depends on the normal ordering. Moreover, the expansion of the entropy in terms of the expectation values (19) yields a temperature dependence. For more details see [18–21].

3. Other applications

Meanwhile the method of flow equations has been applied to many systems. Apart from those already mentioned we list the following applications: the Anderson impurity model [25, 27, 95], Fano-Anderson and Anderson lattice [83], spin-boson models and dissipation [26, 28, 31, 40, 44, 66, 76–78, 92], qubit and the spin-boson model [97], the electron–phonon interaction and superconductivity [32, 34, 49, 52, 88, 116], superconductivity and impurities [39], the boson fermion model [50, 71, 93, 103], the Tomonaga–Luttinger model [79, 94], one-dimensional fermions [85], the Kondo model [36, 70, 84, 87, 106, 109, 113, 119, 121], Fermi and Luttinger liquid [35, 73], the sine-Gordon model [56, 69], QED [29, 30, 43], QCD and general [24, 51, 62, 68, 75, 99, 104], two-dimensional δ-potential [37, 38, 55, 67, 89], limit cycles and three-body problem [82, 96], mapping of the Hubbard Model [33, 107], Heisenberg anti-ferromagnet [45, 59], spin-Peierls transition [48], spin models [58, 72, 74, 86, 90, 91, 100, 102, 105, 110, 117], RKKY interaction [54], heavy fermions [111], interacting Bosons [118, 120], the Lipkin model [44, 46, 60, 98, 101, 115], the Lipkin–Meshkov–Glick model [112, 114], Dirac particle [47], molecules [81], the Henon–Heiles Hamiltonian [53], quartic oscillator [108], complex eigenvalues [80]. It may be mentioned that the two-beam coupling in photorefractive media itself obeys the flow equation scheme [57].

The author gave short reviews on the flow equation method at several occasions [41, 42, 61, 63, 64] mainly explaining the elimination of the electron–phonon interaction and the application to an n-orbital model in the limit of large n, based mainly on [12, 16]. Stefan Kehrein prepares a book on the flow equation approach to many-body problems [122].

4. Symmetry breaking

In applying flow equations to a Hamiltonian one typically starts out from a Hamiltonian which does not show an explicit symmetry breaking even if the symmetry will be broken below some temperature.

The same applies for the renormalization group flows. As indicated above, in the case of flow equations one can bring the effective interaction to the molecular-field form and finally apply molecular field theory. Since we transform the interaction no divergencies appear. In the case of the fermionic renormalization group, the vertex functions will at least within weak-coupling approximations diverge at some length scale, so that the approximations become unreliable and one has to resort to other methods in this regime. Thus, it is desirable to have a way to introduce symmetry breaking from the beginning. Recently Salmhofer, Honerkamp, Metzner and Lauscher [22] have added a symmetry breaking field to the Hamiltonian and showed that this leads into the symmetry broken phase.

For the Hamiltonian flow, it is not necessary to add a symmetry breaking term to the Hamiltonian. Instead, it is sufficient to choose a normal-ordering which is symmetry broken. One can show [23] that the system will nearly always converge to the stable state, that is, in the case of symmetry breaking (that is below the critical temperature) it runs to a symmetry broken state, whereas if the symmetric state is stable (above T_c) then it will run to the symmetric state. The basic idea is the following: normal ordering is given by the bilinear expectation values

$$G_{kj} = \langle a_k a_j \rangle, \tag{20}$$

where the a_k stand for creation and annihilation operators $c_k^\dagger$ and c_k, respectively. In general, one will have normal and anomal expectation values. For given expectation values G one obtains from an operator A the normal-ordered one

$$: A_G :_G = A \tag{21}$$

by

$$A_G = \exp\left(\sum_{kj} G_{kj} \frac{\partial^2}{\partial a_j^{\text{right}} \partial a_k^{\text{left}}}\right) A(a). \tag{22}$$

In this expression the operators a anticommute for fermions. Summation runs over all pairs a, where a_j is to the right of a_k. An infinitesimal change in G leads to a change of A_G

$$\delta A_G = A_{G+\delta G} - A_G = \frac{1}{2} \sum_{kj} \delta G_{kj} \frac{\partial^2}{\partial a_j \partial a_k} A_G. \tag{23}$$

Thus under the Hamiltonian flow, the Hamiltonian H_G will change due to the unitary transformation and due to the change of normal ordering yielding

$$\frac{\mathrm{d}H_G}{\mathrm{d}\ell} = [\eta, H_G] + \frac{\delta H_G}{\delta G} \frac{\partial G}{\partial \ell}. \tag{24}$$

As expectation values G entering the normal ordering we may use the expectation values of $\exp(-\beta H^0)$ with the one-particle Hamiltonian

$$H^0 = \sum_{kj} \frac{1}{2} \tilde{\epsilon}_{kj} a_k^\dagger a_j. \tag{25}$$

Besides the flow equation for the Hamiltonian, a flow equation for H^0 has to be introduced. This can be done by requiring that $\tilde{\epsilon}$ approaches the one-particle part of H_G,

$$H_G = v^{(0)} + \frac{1}{2} \sum_{kj} v_{k^*j}^{(1)} a_k a_j + O(a^4), \tag{26}$$

where $a_{k*} = a_k^\dagger$ by means of the equation

$$\frac{\partial \tilde{\epsilon}_{kj}}{\partial \ell} = \gamma \left(v^{(1)}_{k^*j} - \tilde{\epsilon}_{kj} \right) \tag{27}$$

with some positive constant γ.

Under the following assumptions, one can show [23] that the flow approaches a stable phase: to show this we determine the free energy with respect to the density operator $\exp(-\beta H^0)/Z^0$. If this free energy

$$\begin{aligned} F^0 &= \langle H \rangle_0 - TS, \qquad \langle H \rangle_0 = v^{(0)}, \\ S &= -\frac{k_B}{2} \operatorname{tr}(\tilde{G} \ln \tilde{G} + (1-\tilde{G}) \ln(1-\tilde{G})), \qquad \tilde{G}_{kj} = G_{k^*j} \end{aligned} \tag{28}$$

is a local minimum against variation of H^0, then the fixed point of $H(\infty)$ is stable; if it is not a local minimum then it is unstable. Without performing the flow, that is for $\eta \equiv 0$, this procedure approaches nearly always the Hartree–Fock–Bogoliubov solution.

References

[1] Stueckelberg E C G and Petermann A 1953 La normalisation des constantes dans la theorie des quanta *Helv. Phys. Acta* **26** 499

[2] Gell-Mann M and Low F E 1954 Quantum electrodynamics at small distances *Phys. Rev.* **95** 1300

[3] Kadanoff L P 1965 Scaling laws for Ising models near T_c *Physics* **2** 263

[4] Wilson K G 1971 Renormalization group and critical phenomena. I: renormalization group and the Kadanoff scaling picture. II: phase-space cell analysis of critical behavior *Phys. Rev.* B **4** 3174, 3184

[5] Shankar R 1991 Renormalization group for interacting fermions in $d > 1$ *Physica* A **177** 530

[6] Shankar R 1994 Renormalization group approach to interacting fermions *Rev. Mod. Phys.* **66** 129–92 (*Preprint* cond-mat/9307009)

[7] Zanchi D and Schulz H J 2000 Weakly correlated electrons on a square lattice: renormalization group theory *Phys. Rev.* B **61** 13609 (*Preprint* cond-mat/9812303)

[8] Halboth C J and Metzner W 2000 Renormalization group analysis of the two-dimensional Hubbard model *Phys. Rev.* B **61** 7364 (*Preprint* cond-mat/9908471)

[9] Salmhofer M and Honerkamp C 2001 Fermionic renormalization group flows—technique and theory *Prog. Theor. Phys.* **105** 1

[10] Głazek S D and Wilson K G 1993 Renormalization of Hamiltonians *Phys. Rev.* D **48** 5863

[11] Głazek S D and Wilson K G 1994 Perturbative renormalization group for Hamiltonians *Phys. Rev.* D **49** 4214

[12] Wegner F 1994 Flow equations for Hamiltonians *Ann. Phys., Lpz.* **3** 77

[13] Chu M T and Driessel K R 1990 The projected gradient method for least square matrix approximations with spectral constraints *SIAM J. Numer. Anal.* **27** 1050–60

[14] Brockett R W 1991 Dynamical systems that sort lists, diagonalize matrices, and solve linear programming problems *Linear Algebra Appl.* **146** 79–91

[15] Chu M T 1994 A list of matrix flows with applications *Fields Inst. Commun.* **3** 87–97

[16] Lenz P and Wegner F 1996 Flow equations for electron–phonon interactions *Nucl. Phys.* B **482** 693–712 (*Preprint* cond-mat/9604087)

[17] Thimmel B 1996 Flussgleichungen für das Kondo–Modell *Diploma Thesis* Heidelberg

[18] Grote I, Körding E and Wegner F 2002 Stability analysis of the Hubbard model *J. Low Temp. Phys.* **126** 1385 (*Preprint* cond-mat/0106604)

[19] Hankevych V, Grote I and Wegner F 2002 Pomeranchuk and other instabilities in the *t-t*' Hubbard model at the Van Hove filling *Phys. Rev.* B **66** 094516 (*Preprint* cond-mat/0205213)

[20] Hankevych V and Wegner F 2003 Superconductivity and instabilities in the *t-t*' Hubbard model *Acta Phys. Pol.* B **34** 497

Hankevych V and Wegner F 2003 Superconductivity and instabilities in the *t-t*' Hubbard model *Acta Phys. Pol.* B **34** 1591 (*Preprint* cond-mat/0205597) erratum

[21] Hankevych V and Wegner F 2003 Possible phases of the two-dimensional *t-t*' Hubbard model *Eur. Phys. J.* B **31** 333 (*Preprint* cond-mat/0207612)

[22] Salmhofer M, Honerkamp C, Metzner W and Lauscher O 2004 Renormalization group flows into phases with broken symmetry *Prog. Theor. Phys.* **112** 943–70 (*Preprint* cond-mat/0409725)

[23] Körding E and Wegner F 2006 Flow equations and normal ordering *J. Phys. A: Math. Gen.* **39** 1231–7 (*Preprint* cond-mat/0509801)

[24] Wilson K G, Walhout T S, Harindranath A, Zhang W M, Perry R J and Glazek S D 1994 A weak-coupling treatment on the light front *Phys. Rev.* D **49** 6720 (*Preprint* hep-th/9401153)

[25] Kehrein S K and Mielke A 1994 Flow equations for the Anderson Hamiltonian *J. Phys. A: Math. Gen.* **27** 4259–79

Kehrein S K and Mielke A 1994 Flow equations for the Anderson Hamiltonian *J. Phys. A: Math. Gen.* **27** 5705 (*Preprint* cond-mat/9405034) corrigendum

[26] Kehrein S K, Mielke A and Neu P 1996 Flow equations for the spin-boson problem *Z. Phys.* B **99** 269

[27] Kehrein S K and Mielke A 1996 Theory of the Anderson impurity model: the Schrieffer–Wolff transformation re-examined *Ann. Phys., NY* **252** 1 (*Preprint* cond-mat/9510145)

[28] Kehrein S K and Mielke A 1996 On the spin-boson model with a sub-Ohmic bath *Phys. Lett.* A **219** 313 (*Preprint* cond-mat/9602029)

[29] Brisudova M and Perry R 1996 Initial bound state studies in light-front QCD *Phys. Rev.* D **54** 1831 (*Preprint* hep-ph/9511443)

[30] Jones B D, Perry R and Glazek S D 1997 Nonperturbative QED: an analytical treatment on the light front *Phys. Rev.* D **55** 6561 (*Preprint* hep-th/9605231)

[31] Kehrein S K and Mielke A 1997 Low temperature equilibrium correlation functions in dissipative quantum systems *Ann. Phys., Lpz.* **6** 90 (*Preprint* 9607160)

[32] Mielke A 1997 Similarity renormalization of the electron–phonon coupling *Ann. Phys., Lpz.* **6** 215–33 (*Preprint* cond-mat/9609065)

[33] Stein J 1997 Flow equations and the strong-coupling expansion for the Hubbard model *J. Stat. Phys.* **88** 487

[34] Mielke A 1997 Calculating critical temperatures of superconductivity from a renormalized hamiltonian *Europhys. Lett.* **40** 195–200 (*Preprint* cond-mat/9709175)

[35] Kabel A and Wegner F 1997 Flow equations for Hamiltonians: crossover from Luttinger to Landau-liquid behaviour in the n-orbital model *Z. Phys.* B **103** 555

[36] Vogel E 1997 Flussgleichungen für das Kondo–Modell *Diploma Thesis* Heidelberg

[37] Głazek S D 1997 Renormalization of Hamiltonians *Lecture Notes of the First International School on Light-Cone Quantization* (Ames, Io: Iowa State University Press) (*Preprint* hep-th/9706149)

[38] Głazek S D and Wilson K G 1998 Asymptotic freedom and bound states in Hamiltonian dynamics *Phys. Rev.* D **57** 3558 (*Preprint* hep-th/9707028)

[39] Crisan M, Moca C P and Tifrea I 1997 The flow-equation method for the single impurity in a 2D superconductor *J. Supercond.* **10** 251

[40] Kehrein S K and Mielke A 1998 Diagonalization of system plus environment Hamiltonians *J. Stat. Phys.* **90** 889–98 (*Preprint* cond-mat/9701123)

[41] Wegner F 1998 Flow equations for Hamiltonians *Phil. Mag.* B **77** 1249

[42] Wegner F 1998 Hamiltonian flow in condensed matter physics *New Non-Perturbative Methods and Quantization on the Light Cone* vol 8 ed M Grangé *et al* Les Houches School 1997 (Berlin: Editions de Physique/Springer) p 33

[43] Gubankova E L and Wegner F 1998 Flow equations for QED in the light front dynamics *Phys. Rev.* D **58** 025012 (*Preprint* hep-th/9710233)

[44] Mielke A 1998 Flow equations for band-matrices *Eur. Phys. J.* B **5** 605–11 (*Preprint* quant-ph/9803040)

[45] Stein J 1998 Flow equations and extended Bogoliubov transformation for the Heisenberg antiferromagnet near the classical limit *Eur. Phys. J.* B **5** 193

[46] Pirner H J and Friman B 1998 Hamiltonian flow equations for the Lipkin model *Phys. Lett.* B **434** 231 (*Preprint* nucl-th/9804039)

[47] Bylev A B and Pirner H J 1998 Hamiltonian flow equations for a Dirac particle in an external potential *Phys. Lett.* B **428** 329 (*Preprint* hep-th/9712203)

[48] Uhrig G S 1998 Nonadiabatic approach to spin-Peierls transitions via flow equations *Phys. Rev.* B **57** R14004 (*Preprint* cond-mat/9801185)

[49] Moca C P, Crisan M and Tifrea I 1998 Flow-equation method for a superconductor with magnetic correlations *J. Supercond.* **11** 719 (*Preprint* cond-mat/9806326)

[50] Moca C P, Tifrea I and Crisan M 1999 Flow-equations for the model of hybridized bosons and fermions *J. Supercond.* **12** 399 (*Preprint* cond-mat/9806327)

[51] Walhout T S 1999 Similarity renormalization, Hamiltonian flow equations, and Dyson's intermediate representation *Phys. Rev.* D **59** 065009 (*Preprint* hep-th/9806097)

[52] Ragwitz M and Wegner F 1999 Flow equations for electron–phonon interactions: phonon damping *Eur. Phys. J.* B **8** 9

[53] Cremers D and Mielke A 1999 Flow equations for the Henon-Heiles Hamiltonian *Physica* D **126** 123–35 (*Preprint* quant-ph/9809086)
[54] Stein J 1999 Flow equations and the Ruderman–Kittel–Kasuya–Yosida interaction *Eur. Phys. J.* B **12** 5
[55] Szpigel S and Perry R J 1999 A renormalization group for Hamiltonian field theory *Proc. 1998 YITP-Workshop on the Physics of Hadrons and QCD* ed H Yabu, K Itakura, T Matsui and M Oka (Singapore: World Scientific) (*Preprint* nucl-th/9901079)
[56] Kehrein S 1999 Flow equation solution for the weak to strong-coupling crossover in the sine-Gordon model *Phys. Rev. Lett.* **83** 4914 (*Preprint* cond-mat/9908048)
[57] Anderson D Z, Brockett R W and Nuttall N 1999 Information dynamics of photorefractive two-beam coupling *Phys. Rev. Lett.* **82** 1418
[58] Knetter C and Uhrig G S 2000 Perturbation theory by flow equations: dimerized and frustrated $S = 1/2$ chain *Eur. Phys. J.* B **13** 209 (*Preprint* cond-mat/9906243)
[59] Stein J 2000 Flow equations and new weak-coupling solution for the spin-polaron in a quantum antiferromagnet *Europhys. Lett.* **50** 68
[60] Stein J 2000 Unitary flow of the bosonized large-N Lipkin model *J. Phys. G: Nucl. Part. Phys.* **26** 377
[61] Wegner F 2001 Flow equations for Hamiltonians *Renormalization Group Theory in the new Millenium II (Proceedings of the RG 2000 in Taxco, Mexico) Phys. Rep.* **348** 77
[62] Pauli H C 2000 Applying the flow equations to QCD *Nucl. Phys. B: Proc. Suppl.* **90** 147
[63] Wegner F 2000 Flow equations for Hamiltonians *Non-perturbative QCD and hadron phenomenology* ed H C Pauli and L C L Hollenberg *Nucl. Phys. B: Proc. Suppl.* **90** 141
[64] Wegner F 2000 Flow equations for Hamiltonians *Advances in Solid State Physics* **40** 113
[65] Wegner F 2001 Flow equations for Hamiltonians *Proc. 2nd Conference on the Exact Renormalization Group (Rome 2000)* ed S Arnone, Y A Kubyshin, T R Morris and K Yoshida *Int. J. Mod. Phys.* A **16** 1941
[66] Mielke A 2000 Diagonalization of dissipative quantum systems: I. exact solution of the spin-boson model with an Ohmic bath at alpha=1/2 *Preprint*
[67] Szpigel S and Perry R J 2000 The similarity renormalization group *Quantum Field Theory. A 20th Century Profile* (*Preprint* hep-ph/0009071)
[68] Głazek S D 2000 Running couplings in Hamiltonians *Acta Phys. Pol.* B **31** 909 (*Preprint* hep-th/0001042)
[69] Kehrein S 2001 Flow equation approach to the sine-Gordon model *Nucl. Phys.* B **592** 512–62 (*Preprint* cond-mat/0006403)
[70] Hofstetter W and Kehrein S 2001 Flow equation analysis of the anisotropic Kondo model *Phys. Rev.* B **63** 140402 (*Preprint* cond-mat/0008242)
[71] Domanski T and Ranninger J 2001 Non-linear feedback effects in coupled boson-fermion system *Phys. Rev.* B **63** 134505 (*Preprint* cond-mat/0012081)
[72] Raas C, Bühler A and Uhrig G S 2001 Effective spin models for spin-phonon chains by flow equations *Eur. Phys. J.* B **21** 369 (*Preprint* cond-mat/0102398)
[73] Heidbrink C P and Uhrig G S 2002 Landau's quasi-particle mapping: fermi liquid approach and Luttinger liquid behavior *Phys. Rev. Lett.* **88** 146401 (*Preprint* cond-mat/0111078)
[74] Brenig W and Honecker A 2002 Planar pyrochlore: a strong-coupling analysis *Phys. Rev.* B **65** 140407 (*Preprint* cond-mat/0111405)
[75] Głazek S D 2001 Dynamics of effective gluons *Phys. Rev.* D **63** 116006 (*Preprint* hep-th/0012012)
[76] Stauber T and Mielke A 2002 Equilibrium correlation functions of the spin-boson model with sub-Ohmic bath *Phys. Lett.* A **305** 275 (*Preprint* cond-mat/0207414)
[77] Stauber T and Mielke A 2003 Contrasting different flow equations for a numerically solvable model *J. Phys. A: Math. Gen.* **36** 2707 (*Preprint* cond-mat/0209643)
[78] Stauber T 2003 Universal asymptotic behavior in flow equations of dissipative systems *Phys. Rev.* B **68** 125102 (*Preprint* cond-mat/0211596)
[79] Stauber T 2003 Tomonaga–Luttinger model with impurity at weak two-body interaction *Phys. Rev.* B **67** 205107 (*Preprint* cond-mat/0211598)
[80] Ohira Y and Imafuku K 2002 Flow equation approach to diagonal representation of an unbounded Hamiltonian with complex eigenvalues *Phys. Lett.* A **293** 223 (*Preprint* quant-ph/0201005)
[81] White S R 2002 A numerical canonical transformation approach to quantum many body problems *J. Chem. Phys.* **117** 7472 (*Preprint* cond-mat/0201346)
[82] Głazek S D and Wilson K G 2002 Limit cycles in quantum theories *Phys. Rev. Lett.* **89** 230401 (*Preprint* hep-th/0203088)
[83] Becker K W, Huebsch A and Sommer T 2002 Renormalization approach to many-particle systems *Phys. Rev.* B **66** 235115 (*Preprint* cond-mat/0208351)

[84] Kehrein S and Vojta M 2002 Soliton Fermi sea in models of Ising-coupled Kondo impurities *Preprint* cond-mat/0208390 (extended version published as [106])

[85] Heidbrink C P and Uhrig G S 2002 Renormalization by continuous unitary transformations: one-dimensional spinless fermions *Eur. Phys. J.* B **30** 443 (*Preprint* cond-mat/0208446)

[86] Brenig W 2003 Spin dynamics of a tetrahedral cluster magnet *Phys. Rev.* B **67** 064402 (*Preprint* cond-mat/0208472)

[87] Slezak C, Kehrein S, Pruschke Th and Jarrell M 2003 Semi-analytical solution of the Kondo model in a magnetic field *Phys. Rev.* B **67** 184408 (*Preprint* cond-mat/0208539)

[88] Huebsch A and Becker K W 2003 Renormalization of the electron–phonon interaction: a reformulation of the BCS-gap equation *Eur. Phys. J.* **33** 391 (*Preprint* cond-mat/0209311)

[89] Głazek S D and Mlynik J 2003 Optimization of perturbative similarity renormalization group for Hamiltonians with asymptotic freedom and bound states *Phys. Rev.* D **67** 034019 (*Preprint* hep-th/0210110)

[90] Schmidt K-P, Monien H and Uhrig G S 2003 Rung-singlet phase of the $S = 1/2$ two-leg spin-ladder with four-spin cyclic exchange *Phys. Rev.* B **67** 184413 (*Preprint* cond-mat/0211429)

[91] Schmidt K P and Uhrig G S 2003 Excitations in one-dimensional $S = 1/2$ quantum antiferromagnets *Phys. Rev. Lett.* **90** 227204 (*Preprint* cond-mat/0211627)

[92] Kleff S, Kehrein S and Delft J V 2003 Flow equation renormalization of a spin-boson model with a structured bath *Physica* E **18** 343 (*Preprint* cond-mat/0302357)

[93] Domanski T 2003 Feshbach resonance described by the boson fermion coupling *Phys. Rev.* A **68** 013603 (*Preprint* cond-mat/0302406)

[94] Stauber T 2004 One-dimensional conductance through an arbitrary delta impurity *Phys. Rev.* B **69** 113315 (*Preprint* cond-mat/0301586)

[95] Stauber T and Guinea F 2004 Assisted hopping in the Anderson impurity model: a flow equation study *Phys. Rev.* B **69** 035301 (*Preprint* cond-mat/0303168)

[96] Głazek S D and Wilson K G 2004 Universality, marginal operators, and limit cycles *Phys. Rev.* B **69** 094303 (*Preprint* cond-mat/0303297)

[97] Kleff S, Kehrein S and von Delft J 2004 Exploiting environmental resonances to enhance qubit quality factors *Phys. Rev.* B **70** 014516 (*Preprint* cond-mat/0304177)

[98] Bartlett B H Flow equations for Hamiltonians from continuous unitary transformations *Master's Thesis* (*Preprint* nucl-th/0305052)

[99] Głazek S D and Mlynik J 2004 Accuracy estimate for a relativistic Hamiltonian approach to bound-state problems in theories with asymptotic freedom *Acta Phys. Pol.* B **35** 723 (*Preprint* hep-th/0307207)

[100] Schmidt K P, Knetter Ch and Uhrig G S 2004 Spectral properties of the dimerized and frustrated $S = 1/2$ chain *Phys. Rev.* B **69** 104417 (*Preprint* cond-mat/0307678)

[101] Scholtz F G, Bartlett B H and Geyer H B 2003 Nonperturbative flow equations from running expectation values *Phys. Rev. Lett.* **91** 080602

[102] Knetter Ch and Uhrig G S 2004 Dynamic structure factor of the two-dimensional Shastry–Sutherland model *Phys. Rev. Lett.* **92** 027204 (*Preprint* cond-mat/0309408)

[103] Domanski T and Ranninger J 2003 Bogoliubov shadow bands in the normal state of superconducting systems with strong pair fluctuations *Phys. Rev. Lett.* **91** 255301 (*Preprint* cond-mat/0309468)

[104] Głazek S D 2003 Similarity renormalization group as a theory of effective particles *Proceedings of Light-Cone Workshop: Hadrons and Beyond (LC 03)* (Durham, England) ed S Dalley (*Preprint* hep-th/0310248)

[105] Knetter Ch, Schmidt K P and Uhrig G S 2004 High order perturbation theory for spectral densities of multi-particle excitations: $S = 1/2$ two-leg Heisenberg ladder *Eur. Phys. J.* B **36** 525 (*Preprint* cond-mat/0312246)

[106] Garst M, Kehrein S, Pruschke T, Rosch A and Vojta M 2004 Quantum phase transition of Ising-coupled Kondo impurities *Phys. Rev.* B **69** 214413

[107] Reischl A, Müller-Hartmann E and Uhrig G S 2004 Systematic mapping of the Hubbard model to the generalized t-J model *Phys. Rev.* B **70** 245124 (*Preprint* cond-mat/0401028)

[108] Dusuel S and Uhrig G S 2004 The quartic oscillator: a non-perturbative study by continuous unitary transformations *J. Phys. A: Math. Gen.* **37** 9275 (*Preprint* cond-mat/0405166)

[109] Lobaskin D and Kehrein S 2005 Crossover from non-equilibrium to equilibrium behavior in the time-dependent Kondo model *Phys. Rev.* B **71** 193303 (*Preprint* cond-mat/0405193)

[110] Sykora S, Huebsch A and Becker K W 2004 Computer aided perturbation theory by cumulants: dimerized and frustrated spin 1/2 chain *Phys. Rev.* B **70** 054408 (*Preprint* cond-mat/0405420)

[111] Meyer K 2004 Flow equation approach to heavy fermion systems *Eur. Phys. J.* B **42** 17 (*Preprint* cond-mat/0406264)

[112] Dusuel S and Vidal J 2004 Finite-size scaling exponents of the Lipkin–Meshkov-Glick model *Phys. Rev. Lett.* **93** 237204 (*Preprint* cond-mat/0408624)

[113] Kehrein S K 2005 Scaling and decoherence in the out-of-equilibrium Kondo model *Phys. Rev. Lett.* **95** 056602 (*Preprint* cond-mat/0410341)

[114] Dusuel S and Vidal J 2005 Continuous unitary transformations and finite-size scaling exponents in the Lipkin-Meshkov-Glick model *Phys. Rev.* B **71** 224420 (*Preprint* cond-mat/0412127)

[115] Kriel J N, Morozov A Y and Scholtz F G 2005 Non-perturbative flow equations from continuous unitary transformations *J. Phys. A: Math. Gen.* **38** 205 (*Preprint* cond-mat/0408420)

[116] Dusuel S and Vidal J 2005 Finite-size scaling exponents and entanglement in the two-level BCS model *Phys. Rev.* A **71** 060304 (*Preprint* cond-mat/0501282)

[117] Uhrig G S, Schmidt K P and Grüninger M 2005 Magnetic excitations in bilayer high-temperature superconductors with stripe correlations *J. Phys. Soc. Japan* (suppl.) **74** 86 (*Preprint* cond-mat/0502460)

[118] Dusuel S, Vidal J, Arias J M, Dukelsky J and Garcia-Ramos J E 2005 Finite-size scaling exponents in the interacting boson model *Phys. Rev.* C **72** 011301 (*Preprint* cond-mat/0503240)

[119] Sommer T 2005 Flow equations for the one-dimensional Kondo lattice model: static and dynamic ground state Properties (*Preprint* cond-mat/0503423)

[120] Reischl A, Schmidt K P and Uhrig G S 2005 Temperature in one-dimensional bosonic Mott insulators *Phys. Rev.* A **72** 063609 (*Preprint* cond-mat/05047240)

[121] Vogel E 2005 Flussgleichungen für das Kondo–Modell *PhD Thesis* Heidelberg

[122] Kehrein S 2006 The flow equation approach to many-particle physics *Springer Tracts Modern Phys.* at press